

Chapter 1

Dutch Strategies, Accuracy-Dominance, All-or-Nothing Beliefs, and their Plannings

1.1 Abstract

Suppose we consider an agent with both numerical credences and all-or-nothing beliefs. This agent might also have a plan about how she is going to update her beliefs upon receiving new evidence. What rational requirements on such a plan can be justified from an epistemic value point of view? Plan Almost Lockean Revision is the claim that it is rationally required that one's planned beliefs are exactly one's sufficiently high conditional credences. We start by reviewing arguments available for Plan Almost Lockean Revision in the current literature, ultimately concluding that they are non-optimal. We provide a better argument to the effect that the belief updating rule that is expected to be the best according to one's current credences is exactly Plan Almost Lockean Revision, that is, we prove a qualitative version of the Greaves-Wallace Theorem that Plan Conditionalization maximizes expected accuracy according to one's current credences. Furthermore, building on the work of (Rothschild, 2021), we investigate the dutchbookability of Lockean betting behavior for all-or-nothing beliefs and their plannings, ultimately proving a qualitative version of the dutch strategy theorem which leads to the development of novel dutch-strategy/accuracy-dominance arguments for Lockean norms on belief/belief-planning pairs.

1.2 The Almost Lockean Thesis

Suppose we consider an agent with both numerical credences and all-or-nothing beliefs on the same collection of finitely-many propositions. We can thus represent the agent's doxastic state (at time t) as a pair (B_t, c_t) ¹, consisting of her set of beliefs

¹Henceforth, we drop the time indices. All unindexed pairs are considered synchronic. Furthermore, we use the symbol ' t ' to refer to something else entirely. It should be clear from the context to what ' t ' is referring.

and her credences. Question: How must one's credences and beliefs be related in order for them to rationally cohere? Kyburg (1974) and Foley (1992) suggested the following answer: one's beliefs must be exactly one's high credences. This is called the Almost Lockean Thesis². So, according to the Almost Lockean Thesis, if you believe that Amy is coming to your party tonight, then you should have high enough confidence that Amy is coming to your party tonight and, if you have high enough confidence that Amy is coming to your party tonight, then you should believe that Amy is coming to your party tonight. More precisely,

Almost Lockean Thesis (with threshold $t > \frac{1}{2}$): (B, c) is such that:

- (1): If $p \in B$, then $c(p) \geq t$.
- (2): If $p \notin B$, then $c(p) \leq t$.

But why think it a requirement of rationality? Dorst (2019), building off the insights of Easwaran (2015) and Hempel (1962), develops an expected-accuracy argument in its favor. Before detailing this argument, we first have to understand how we are going to measure the accuracy of our all-or-nothing beliefs.

1.3 Accuracy for Beliefs and Dorst's Expected-Accuracy Argument

We start with a very basic and plausible idea. It seems good, from an epistemic point of view, to believe true things and bad, from an epistemic point of view, to believe false things. We can formalize this intuition numerically by assigning a (non-negative) real number T as the value of believing true propositions and assigning the (non-positive) real number F as the disvalue of believing false propositions.³ We call this property Extensionality. Precisely,

Def⁴: accuracy-measure⁵ A is said to be Extensional iff

$$A(\{p\}, B, w) = \begin{cases} T & \text{if } p \in B \text{ is true at } w. \\ F & \text{if } p \in B \text{ is false at } w. \\ 0 & \text{if } p \notin B. \end{cases} \quad (1.1)$$

Great, we now have a way to measure the accuracy of your all-or-nothing beliefs, or lack thereof, in particular propositions. But, how are we going to measure the accuracy of your entire belief-set? Answer: we measure it additively over propositions.

²Strictly speaking, our Almost Lockean Thesis differs from (Foley, 1992)'s Lockean Thesis (with threshold t), that $B = \{p : c(p) \geq t\}$, by saying nothing about the belief/disbelief status of propositions at the threshold. Hence the "Almost" qualifier, which we adopt from (Rothschild, 2021).

³The possible worlds, the w 's, are going to be understood as the logically possible disjunctive normal forms of our agent's finitely-many propositions that they have doxastic attitudes towards.

⁴For the sake of simplicity, we are going to ignore including the attitude of suspended judgement in this paper. So, when I write " $p \notin B$ ", I mean that our agent doesn't believe p . See (Dorst, 2019) for some discussion on accuracy and suspended judgement.

⁵Formally, A is a function from the set of triples consisting of a proposition, a belief-set, and a possible world to the set of real numbers.

That is, we look at the value/disvalue of your believing propositions and sum those values over all such propositions. Precisely,

Def: accuracy-measure A is said to be Additive iff the accuracy of belief-set B at world w is such that $A(B, w) = \sum_{p \in B} A(\{p\}, B, w)$.

Finally, our last legitimacy condition on qualitative accuracy-measures that we'll need is as follows:

Def: accuracy-measure A is said to be Variable Conservative iff $T > 0 > F$ and $|F| > T$.

This is the first and only condition that we impose that actually constrains how T and F are related. The first part of Variable Conservativeness says that it is *strictly* good to believe true things and *strictly* bad to believe false things. The second part says that the disvalue of believing false things is greater than the value of believing true things. One motivation for why we might want the second part of Variable Conservativeness is because it helps us rule out the rational permissibility of believing both p and $\neg p$. Because, after all, if $|F| \leq T$, then $A(p \in B, w) + A(\neg p \in B, w) \geq 0$, so why not then believe both p and $\neg p$. Further discussion of this point can be found in (Steinberger, 2019) and (Hewson, 2020).

Great, having finally finished our discussion of how we are going to legitimately measure the accuracy of one's all-or-nothing beliefs, without further ado, here is the key technical result for Dorst's expected-accuracy argument:

Easwaran-Dorst Theorem: Let c be a probabilistic credence function. Let qualitative accuracy-measure A satisfy Additivity, Extensionality, and Variable Conservativeness. Then, belief-set B maximizes expected accuracy⁶ with respect to c iff the pair (B, c) satisfies the Almost Lockean Thesis with threshold $t = \frac{-F}{T-F}$.

In detail, assuming that the relevant credence function is probabilistic, Dorst's expected-accuracy argument for the Almost Lockean Thesis with threshold t is:

⁶In discussion, Teddy Seidenfeld asked why we don't, in analogy with the credal case, require that our qualitative accuracy-measures satisfy a strict propriety condition? I take him to be saying something like: legitimate belief accuracy-measures A are such that for every probability function c there exists a unique B that maximizes c -expected accuracy. Well, it turns out that no such additive A 's exist. Given the Easwaran-Dorst Theorem and Variable Conservativeness, just pick a probability function c such that $Tc(p) + Fc(\neg p) = 0$, so that we can get two distinct belief-sets that both satisfy the Almost Lockean Thesis wrt c . Now, while some might think that this puts pressure on the Additivity constraint, I think differently. It's not clear that every probability function can only rationally cohere with exactly one belief-set. Why think such inter-theoretic coherence so unique? Secondly, the candidate requirement of strict qualitative propriety is not clearly in analogy with strict propriety on credal accuracy-measures. One says that your credences have to think your beliefs the best; the other says that your credences have to think themselves the best, not to mention that the best motivation for strict propriety is truth-directedness plus *weak* propriety (and additivity) (Campbell-Moore & Levinstein, 2021).

- (1): Qualitative Veritism⁷: only qualitative accuracy is intrinsically epistemically valuable for all-or-nothing beliefs.
- (2): Legitimate qualitative accuracy-measures are Extensional, Additive, and Variable Conservative with threshold t .⁸
- (3): Rational (B, c) 's are such that B maximizes expected accuracy with respect to c .
- (4): **Easwaran-Dorst Theorem.**
- (5): Therefore, the Almost Lockean Thesis with threshold t is rationally required.

Now, Dorst goes on to show that Extensionality can be dropped from his theorem entirely. Doing so permits contextual and proposition-wise dependencies for T and F , which results in an Almost Lockean Thesis with variable/contextual thresholds.⁹ Furthermore, he shows that even Additivity can be significantly weakened while still retaining the Almost Lockean Thesis. Here, we extend matters in a different direction: by investigating rational norms for all-or-nothing belief updating. In more detail, how should an agent rationally change their all-or-nothing beliefs upon coming to possess new evidence E ? Let us look at this.

1.4 Plan Almost Lockean Revision

The literature of proposed norms for rational belief updating is extensive¹⁰, but we will be focused on just one proposal. Shear and Fitelson (2018) in their excellent “Two Approaches to Belief Revision” suggest the following dynamics for updating all-or-nothing belief upon coming to possess new evidence E . Namely, they propose the following as a candidate for a requirement of epistemic rationality on belief updating:

Actual Almost Lockean Revision (with threshold $t > \frac{1}{2}$): For (B_{t_1}, c_{t_1}) and (B_{t_2}, c_{t_2}) , if the agent actually receives total evidence E between times t_1 and t_2 and $c_{t_1}(E) > 0$, then:

- (1): If $p \in B_{t_2}$, then $c_{t_1}(p|E) \geq t$.
- (2): If $p \notin B_{t_2}$, then $c_{t_1}(p|E) \leq t$.

⁷Of course, a more neutral term in place of “accuracy-measure” might be epistemic-utility function or scoring rule for which (Dorst, 2019)’s argument for the Almost Lockean Thesis might avoid commitment to Qualitative Veritism and proceed under something like “whatever is of epistemic value, it is only legitimately measured by an Additive, Extensional, and Variable Conservative function.” Nevertheless, in the Jamesian spirit of giving the “first and great[est]” importance to “Believe truth! Shun error!” (James, 1897), I remain sympathetic to Veritism and continue to use the accuracy terminology.

⁸An accuracy-measure is said to have threshold t when $t = \frac{-F}{T-F}$.

⁹This is a very important development. Perhaps believing some truths is more epistemically valuable than believing other truths. Perhaps it is better to believe truths with higher “informational content” as advocated in (Levi, 1967) and more recently by (Dorst & Manderlker, 2021) and (Skipper, 2023). Perhaps such values depend upon its relevance to a question under discussion (Levi, 1967) and (Yalcin, 2016). Even considerations of verisimilitude can be accommodated within this framework.

¹⁰To name just a few: (Alchourrón, Gärdenfors, & Makinson, 1985) (Lin & Kelly, 2012) (Leitgeb, 2017) (Shear & Fitelson, 2018) (Kelly & Lin, 2021) (Mierzewski, 2022) (Goodman & Salow, 2024) (Pearson, 2025) (Wang, 2025) (Mierzewski, 2025).

In words: one's updated beliefs must be exactly one's high enough conditional credences. In the current literature, the best argument for Actual Almost Lockean Revision is the one provided by Shear and Fitelson (2018, pg. 19):

Thus, Lockean revision, as we have explored it, is entailed by the more general norm requiring that agents have belief sets that maximize expected epistemic value at any given time. Assuming conditionalization as the rational procedure for credal updating, the norm entails that Lockean revision is the unique procedure that will guarantee that agents maximize overall expected epistemic value.

In premise form, we have:

- (1): Actual Conditionalization¹¹.
- (2): Almost Lockean Thesis with threshold t .
- (3): Therefore, Actual Almost Lockean Revision with threshold t .

Can we do better than this argument? Clearly, this argument appeals to Actual Conditionalization in the first premise, so, we might ask, is this argument epistemic-value-theoretic through-and-through? In other words, do there exist good epistemic value arguments for Actual Conditionalization? The prevailing answer in the literature seems to be “no” (Pettigrew, 2016, Chapter 15)¹².

In order to do better, we need the notion of a belief plan $\beta : \epsilon \rightarrow \{\text{Belief Sets}\}$ which is a function from an evidential partition to the set of belief-sets. The notion of a belief plan can be interpreted in, at least, three ways:

- (a): The dispositional interpretation: If an agent were to receive total evidence E , then the agent would adopt beliefs β_E .
- (b): The planning interpretation: The agent plans to adopt beliefs β_E upon receiving total evidence E .
- (c): The suppositional interpretation: The agent's suppositional/conditional beliefs are β_E upon supposing exactly that E .

We say a little about each interpretation. The dispositional interpretation is assumedly deterministic along the lines discussed by (Pettigrew, 2020). There is no chance that you might adopt another belief-set different from β_E . Planning is to be understood as some kind of mental state involving intentionality, that is, it is

¹¹Actual Conditionalization says that it is a requirement of epistemic rationality that if you receive exactly evidence E between times t_1 and t_2 , then your credal pair (c_{t_1}, c_{t_2}) is such that if $c_{t_1}(E) > 0$, then $c_{t_2} = c_{t_1}(\cdot|E)$.

¹²Although, see (Gallow, 2019) for an attempted accuracy-theoretic argument for Actual Conditionalization from value-change. (For a better argument see (Rooyakkers, ms).) I thank Professor Pettigrew for suggesting this paper to me. We note here that an entirely analogous change-of-value argument can be developed for Actual Almost Lockean Revision, namely: given the Easwaran-Dorst conditions, the belief set which maximizes $\sum_{w \in E} A(B, w)c(w)$ satisfies the Almost Lockean Thesis with respect to $c(\cdot|E)$ (if defined). Let $B_{c(\cdot|E)}$ satisfy the Almost Lockean Thesis with respect to $c(\cdot|E)$.

Proof: The Easwaran-Dorst Theorem gives us that $\text{Exp}[A(B_{c(\cdot|E)})|c(\cdot|E)] = \sum_{w \in W} A(B_{c(\cdot|E)}, w)c(w|E) = \sum_{w \in E} A(B_{c(\cdot|E)}, w)c(w|E) = \frac{1}{c(E)} \sum_{w \in E} A(B_{c(\cdot|E)}, w)c(w)$ is maximal over all belief sets B and scaling by $\frac{1}{c(E)}$ (a constant not depending upon B) does not affect the expected accuracy ordering. \diamond .

about what you intend to do (or how you intend to be) upon receiving new evidence E . The suppositional interpretation is indicative (as opposed to subjunctive) and is to be understood along the lines discussed by (Eva, Shear, & Fitelson, 2022), minus the part about E being interpreted as evidence received. After all, supposing that E does not amount to possessing evidence that E ; we can and do engage in suppositional reasoning without treating the supposition as evidence that we came to possess.

It is important to note that, while these interpretations are distinct, they are not necessarily competing. Plans can (and often do) go awry and what one plans to believe upon receiving evidence E need not be the same as their beliefs under the supposition that E . Except where explicitly remarked, we take the view that any interpretation is fitting. Great, now, consider the following candidate requirement of epistemic rationality:

Plan Almost Lockean Revision (with threshold $t > \frac{1}{2}$): the pair (β, c) is such that, for every $E \in \epsilon$ with $c(E) > 0$:

- (1): If $p \in \beta_E$, then $c(p|E) \geq t$.
- (2): If $p \notin \beta_E$, then $c(p|E) \leq t$.

In words: one's planned beliefs must be one's high conditional credences on the relevant evidence. But why think it a requirement of rationality? Well, it turns out that we can develop an expected-accuracy argument in its favor.¹³

1.5 Accuracy for Belief-plannings and the Qualitative Greaves-Wallace Theorem

In order to develop our expected-accuracy argument for Plan Almost Lockean Revision, we have to first say how we are going to measure the accuracy of our belief-plannings. Here is how we do that:

Def: $A(\beta, w) = A(\beta_{E_w}, w)$.

In words, the accuracy of your belief-plan at world w is the accuracy of the planned belief-set that you would adopt at world w . In this way, we can reduce the problem of measuring the accuracy of belief-plannings to that of measuring the accuracy of belief-sets. Great, now that we know how to legitimately measure the accuracy of one's all-or-nothing belief-plannings, without further ado, here is the key technical result for our expected-accuracy argument:

Qualitative Greaves-Wallace Theorem¹⁴: Let c be a probabilistic credence function. Suppose accuracy measure A is Additive, Extensional, and Variable Conserva-

¹³In the appendix, we develop another (Shear & Fitelson, 2018)-style argument for Plan Almost Lockean Revision.

¹⁴In the statement of this theorem, and throughout the rest of this paper, we are going to ignore the concerns raised in (Schoenfeld, 2017), as similar concerns arise for our qualitative cases of interest. The relevant adjustments to all the results in this paper are exactly the expected ones.

tive. Then, (β, c) satisfies Plan Almost Lockean Revision iff β maximizes expected accuracy with respect to c .

Proof:

Maximized Expected Accuracy \Rightarrow Plan Almost Lockean Revision:

Assume belief plan β maximizes expected accuracy. Let $p \in \beta_E$. Let $\beta' = \beta$ everywhere on the evidential partition except on E , where $\beta'_E = \beta_E - p$. Thus, $EA(\beta) - EA(\beta')$

$$\begin{aligned} &= \sum_{w \in E} c(w)[A(\beta_E, w) - A(\beta'_E, w)] \geq 0 \\ &= \sum_{w \in E} c(w)[A(p \in \beta_E, w)] \\ &= \sum_{w \in E \cap p} c(w)T + \sum_{w \in E \cap \neg p} c(w)F \\ &= c(E \cap p)T + c(E \cap \neg p)F \geq 0 \\ &\Rightarrow c(p|E)T + c(\neg p|E)F \geq 0 \text{ by dividing both sides by } c(E) > 0. \\ &= c(p|E)T + (1 - c(p|E))F \geq 0 \\ &\Rightarrow c(p|E) \geq \frac{-F}{T-F}. \end{aligned}$$

An analogous argument gives: if $p \notin \beta_E$, then $c(p|E) \leq \frac{-F}{T-F}$.

Plan Almost Lockean Revision \Rightarrow Maximized Expected Accuracy:

Assume Plan Almost Lockean Revision. So, we have:

- 1.) If $p \in \beta_E$, then $c(p|E) \geq \frac{-F}{T-F}$.
- 2.) If $p \notin \beta_E$, then $c(p|E) \leq \frac{-F}{T-F}$.

Now, assume (for contra.) that β doesn't maximize expected accuracy.

Thus, there exists a β' st. $EA(\beta') > EA(\beta)$ and β' maximizes expected accuracy.

$\Rightarrow \exists p$ for some E st.

1*) $[p \in \beta'_E \text{ and } p \notin \beta_E]$ or

2*) $[p \notin \beta'_E \text{ and } p \in \beta_E]$.

Case 1*): We have that $p \in \beta'_E$ and β' maximizes expected accuracy \Rightarrow (by the first part of the proof) $c(p|E) \geq \frac{-F}{T-F}$. Also, by condition 2.) and $p \notin \beta_E$, we have that $c(p|E) \leq \frac{-F}{T-F}$. Thus, $c(p|E) = \frac{-F}{T-F}$.

Case 2*): Similarly, we find $c(p|E) = \frac{-F}{T-F}$.

But $c(p|E) = \frac{-F}{T-F}$ for all p that β' and β disagree about at some $E \Rightarrow EA(\beta') = EA(\beta)$, a contradiction, because

$$\begin{aligned} &EA(\beta') - EA(\beta) \\ &= \sum_{E \in \epsilon} \sum_{w \in E} c(w)[A(\beta'_E - \beta_E, w) - A(\beta_E - \beta'_E, w)] \\ &= \sum_{E \in \epsilon} [\sum_{w \in E} c(w)A(\beta'_E - \beta_E, w) - \sum_{w \in E} c(w)A(\beta_E - \beta'_E, w)] \\ &= 0 \text{ as } \sum_{w \in E} c(w)A(\beta'_E - \beta_E, w) = 0 \text{ and } \sum_{w \in E} c(w)A(\beta_E - \beta'_E, w) = 0 \text{ for every } E \\ &\text{by backtracking the first part of the proof (for all the relevant } p) \text{ starting with } c(p|E) = \frac{-F}{T-F}. \end{aligned}$$

◇¹⁵.

With this technical result in hand, and assuming that the relevant credence function is probabilistic, we can develop our expected-accuracy argument for Plan Almost Lockean Revision:

¹⁵Perhaps the Lockean threshold might depend upon the evidential context, that is, upon which evidence is received. This can be done by having the value of believing truths and epistemic disvalue of believing falsehoods depend on the evidential context. In this case, it is easy enough to show that an analogous theorem to the one above holds by replacing " T " and " F " with " T_E " and " F_E " respectively where appropriate.

- (1): Qualitative Veritism: only qualitative accuracy is intrinsically epistemically valuable for all-or-nothing belief-plannings.
- (2): Legitimate qualitative accuracy-measures are Extensional, Additive, and Variable Conservative with threshold t .
- (3): Rational (β, c) 's are such that β maximizes expected accuracy with respect to c .
- (4): **Qualitative Greaves-Wallace Theorem.**
- (5): Therefore, Plan Almost Lockean Revision with threshold t is rationally required.

1.6 Lockean Betting for All-or-Nothing Beliefs

There is a long and venerable tradition in statistics, decision theory, and philosophy of connecting¹⁶ credences/previsions to betting behavior (Ramsey, 1926) (de Finetti, 1937, 1974) (Lehman, 1955) (Kemeny, 1955) (Hajek, 2005) (Schervish, Seidenfeld, & Kadane, 2009).¹⁷ Only recently have all-or-nothing beliefs been connected to such betting behavior (Rothschild, 2021). We follow Rothschild in describing this kind of Lockean betting.¹⁸ Consider an agent who has all-or-nothing beliefs B , never mind if they also have credences. They might; they might not. If $p \in B$, then the agent will (or ought) to buy a \$1 bet on p for \$ t (or less). And, if $p \notin B$, then the agent will (or ought) to sell a \$1 bet on p for \$ t (or more). Generalizing this to bets with non-negative stakes other than \$1 is straightforward. If $p \in B$, then the agent will (or ought) to buy a \$ $x \geq 0$ bet on p for \$ tx (or less). And, if $p \notin B$, then the agent will (or ought) to sell a \$ $x \geq 0$ bet on p for \$ tx (or more). Now, with the notion of Lockean betting in-hand, we can ask when some belief-set B is strongly dutchbookable at threshold t , that is, when there exists a collection of bets that B is willing (or ought) to buy/sell¹⁹ at threshold t and that, when taken together additively, guarantee a net loss at every possible world. That is, when can a Lockean bettor be bilked out of money? And, furthermore, how is strong dutchbookability related to strong accuracy-dominance²⁰ for Lockean bettors? These questions are answered by (Rothschild, 2021). We quickly rehearse these answers. But first, a very important definition:

Def: B is said to be Almost Lockean Complete at threshold t iff there exists a probabilistic credence function c st. for every proposition p [if $p \in B$, then $c(p) \geq t$ and if $p \notin B$, then $c(p) \leq t$].

¹⁶This connection being descriptive as in de Finetti's Radical Operationalism or normative as in (Christensen, 1996)'s "sanction as fair" view.

¹⁷To name just a few.

¹⁸We note that (de Finetti, 1981)'s worry about truthful credence elicitation due to strategic pricing in a Prevision Game also arises in the context of Lockean betting if we allow varying thresholds between agents. See (Seidenfeld, 2021) for a discussion of strategic pricing in a Prevision Game in the credal case and strategic concerns in a Forecasting Game.

¹⁹If the dutchbook involves both buying and selling, it is said to be a two-way dutchbook. If it just involves buying, then it is said to be a one-way dutchbook. We focus on two-way dutchbookability in this paper. The terminology is (Rothschild, 2021)'s.

²⁰Belief-set B is said to be strongly accuracy-dominated with respect to legitimate accuracy-measure A iff there exists some belief-set B' such that $A(B, w) < A(B', w)$ for every possible world w . In words, B' is more accurate than B at every possible world.

The idea here is that such Almost Lockean Complete beliefs can be represented as being Almost Lockean with respect to some probabilistic credences. It is important to note that Almost Lockean Completeness, or the lack thereof, is a property of one's all-or-nothing beliefs. It has nothing to do with one's credences. This is unlike the Almost Lockean Thesis, which explicitly relates one's all-or-nothing beliefs and their actual credences. Back to (Rothschild, 2021):

Rothschild's Equivalence Theorem²¹: B is two-way strongly dutchbookable at threshold t iff B is strongly accuracy-dominated with respect to some Additive accuracy-measure A with threshold t .

Qualitative Dutch Book Theorem: B avoids two-way strong dutchbookability at threshold t iff B is Almost Lockean Complete at threshold t .

The proof of the Qualitative Dutch Book Theorem makes use of the following very important and technical Lemma found in (Rothschild, 2021) as "Theorem 2". We mention it explicitly because it will also be useful in the proofs of our subsequent theorems.

Farkas-Rothschild Lemma²²:

Let \mathbf{A} be any $m \times n$ matrix, then:

$$(\nexists \vec{x} \geq 0 : \mathbf{A}\vec{x} < 0) \iff (\exists \vec{y} \geq 0 : \vec{y} \neq 0 \text{ and } \vec{y}\mathbf{A} \geq 0).$$

Now, having finished our brief rehearsal of (Rothschild, 2021), we proceed to extend his results to the diachronic-ish case of all-or-nothing belief updating, but first a discussion of: why the "-ish" in "diachronic-ish"?

1.7 Diachronic Dutchbooks and Beliefs

A genuinely diachronic book is a collection of bets offered at different times. For our purposes, the times $t_1 < t_2$ will suffice. Now, in the credal case, it is well-known that rationally requiring the avoidance of strong diachronic-dutchbookability is an exceedingly strong demand; so strong in fact that one cannot rationally change their credences, even if one comes to possess new evidence.²³ Surely, this result refutes rationally requiring the avoidance of strong diachronic-dutchbookability in the credal case. We show that a similar result holds for the all-or-nothing belief

²¹de Finetti (1974) was the first to explicitly investigate the relationship between dutchbookability and accuracy-dominance with respect to the (Brier, 1950)-score in the credal case, which was later implicitly extended to include the class of proper scoring rules by (Savage, 1971). In this sense, Rothschild's Theorem can be understood as a qualitative version of de Finetti's work.

²²The appearance of this lemma in our dutchbookability inquiries is not entirely unexpected because its proof involves the Separating Hyperplane Theorem, which is known to be useful in investigating dutchbookability (Pettigrew, 2020).

²³In detail, it can be shown that a credal pair (c_{t_1}, c_{t_2}) avoids strong diachronic-dutchbookability iff c_{t_1} is probabilistic and $c_{t_1} = c_{t_2}$. See (Pettigrew, 2020) for an especially clear presentation of this result. The idea is that if $c_{t_1} \neq c_{t_2}$, you, as the bookie, could sell a bet on p for higher at one time and buy a bet on p for cheaper at the other time. And, this gives a strong dutchbook against such (c_{t_1}, c_{t_2}) .

case with Lockean betting. In particular, we show that requiring the avoidance of strong diachronic-dutchbookability at threshold $t > \frac{1}{2}$ would prevent Lockean bettors from changing their mind about p in the sense that one cannot rationally believe p at t_1 and then believe $\neg p$ at t_2 , even if one comes to possess new evidence that $\neg p$.²⁴ Let (B_{t_1}, B_{t_2}) involve such a mind change about p . Now, consider the following bets (which are to be sold to our Lockean bettor): a \$1 bet at t_1 on p for \$ t and a \$1 bet at t_2 on $\neg p$ for \$ t . These two bets, taken together, constitute a strong diachronic-dutchbook against such (B_{t_1}, B_{t_2}) . Thus, requiring the avoidance of strong diachronic-dutchbookability is not the way to go in the qualitative case either.

These observations independently motivate the introduction of belief plans as the proper target for dutchbook theorems in the context of investigating Lockean betting behavior; thus making the forthcoming dutchbook arguments properly strategic (Teller, 1973) (Skyrms, 1987) (Lewis, 1999), fundamentally synchronic (van Fraassen, 1989) (Levi, 2002), and “irreducibly modal” (Pettigrew, 2020) just as in the credal case. Such planned beliefs are to be connected, descriptively or normatively, with Lockean betting behavior in the same way that planned credences are connected with betting behavior: via conditional/called-off bets. In detail, a conditional bet gives a net payoff of \$0 if the condition²⁵ is not fulfilled and works like a normal bet otherwise. The notion of conditional Lockean betting is now immediate. If $p \in \beta_E$, then the agent will (or ought) to buy a \$1 E -conditional bet on p for \$ t (or less). And, if $p \notin \beta_E$, then the agent will (or ought) to sell a \$1 E -conditional bet on p for \$ t (or more). Generalizing this to conditional bets with non-negative stakes other than \$1 is straightforward.

Now, suppose we consider an agent with both all-or-nothing beliefs and their planings. Never mind if our agent has credences or credal planings. They might; they might not. We can thus represent such an agent’s doxastic state with a pair (B, β) . With the notions of Lockean betting and conditional Lockean betting in hand, we can investigate when such Lockean bettors can be bilked out of money, and, furthermore, how the strong dutchbookability of pairs (B, β) is related to strong accuracy-dominance. We begin with the latter question and answer the former in the next section. But first, we must discuss how we are going to measure the accuracy of belief/belief-planning pairs.

Def: accuracy-measure A is said to be Fully-Additive iff $A[(B, \beta), w] = A(B, w) + A_{E_w}(\beta_{E_w}, w)$ and the A and A_e are additive.²⁶

²⁴Demonstrating this is all we need to unseat the standard of avoiding strong diachronic-dutchbookability. However, for the curious, see the Appendix for a full characterization of avoiding two-way strong diachronic-dutchbookability in the qualitative case.

²⁵How, exactly, the relevant condition is to be understood depends upon how one understands belief plans. The condition might be “coming to possess evidence E ”, “supposing that E ”, “ E being true”, etc.

²⁶We stress that A ’s being Fully-Additive does *not* imply that it is then Extensional. It might be that $A(\{p\}, B, w) \neq A_{E_w}(\{p\}, \beta_E, w)$. This observation matters in the proof of the Qualitative SSK Theorem. Its origin lies in the fact that we allow varying stakes between plain and conditional bets on the same proposition. At first face, this permissiveness might seem objectionable. After all, aren’t we comparing the same type of synchronic attitudes using different evaluative standards? No, we aren’t. Recall that we are comparing belief/belief-planning pairs. We’re never, for example,

In words, an accuracy-measure being Fully-Additive means that the accuracy of a belief/belief-planning pair at world w is the sum of the accuracy of one's beliefs at world w and the accuracy of your planned beliefs at world w . And, further, that the accuracy-measure of your beliefs and their plannings is also additive. In this way, Full-Additivity allows us to jointly measure the accuracy of such (B, β) pairings.

Qualitative SSK Theorem²⁷: (B, β) is two-way strongly dutchbookable at threshold t iff (B, β) is strongly accuracy-dominated with respect to some Fully-Additive accuracy-measure A with threshold t .

Proof: It is almost entirely analogous to the proof of **Rothschild's Equivalence Theorem** for the case of just beliefs. In more detail, $A[(B, \beta), w] = A(B, w) + A_{E_w}(\beta_{E_w}, w)$ and Rothschild showed that a stake vector of bets on propositions for a belief-set determines and is determined by a qualitative accuracy-measure. In the same way, a stake vector of all the E -conditional bets on propositions determines and is determined by an accuracy-measure on all-or-nothing belief-plans because measuring the accuracy of a belief-plan at a world reduces to measuring the accuracy of one's planned beliefs at that world.

◇.

1.8 The Qualitative Dutch Strategy Theorem

In this section, we develop the promised dutch strategy theorem for pairs of all-or-nothing beliefs and their plannings, the so-called Qualitative Dutch Strategy Theorem. But first, a few definitions.

Def: belief plan β is said to be Almost Lockean Complete at threshold t iff there exists a probabilistic credence function c st. for every proposition p and for every piece of evidence E if $c(E) > 0$, then [if $p \in \beta_E$, then $c(p|E) \geq t$ and if $p \notin \beta_E$, then $c(p|E) \leq t$].

Def: the pair (B, β) is said to be jointly Almost Lockean Complete at threshold t iff B is Almost Lockean Complete at threshold t with respect to c and β is Almost Lockean Complete at threshold t with respect to the same c . The idea being that there exists a c that “works for both”.

Given this notion of Joint Almost Lockean Completeness, a question immediately presents itself. Is Joint Almost Lockean Completeness substantive, in the sense that it rules out belief/belief-planning pairs that are both individually Almost Lockean Complete? It turns out that the answer is yes! Here's a sketch of how the proof would go. Pick some B such that B contains $\neg p \& q$ (with p and q logically indepen-

comparing your beliefs at time t_1 with some other beliefs at time t_1 using different epistemic standards, that is, using different accuracy-measures.

²⁷SSK refers to (Schervish, Seidenfeld, & Kadane, 2009) who, among other things, investigate when strong dutchbookability (their *incoherence*₁) and scoring-dominance (their *incoherence*₃) coincide in a very general setting involving infinitely-many and even randomized marginal/conditional forecasts.

dent) but doesn't contain q and is Almost Lockean Complete (at threshold $t < 1$). This implies that any Almost Lockean representer of B is such that $c(p) = 1$ (and which implies that you believe p). Now, pick an evidential partition with an $E \in B$ that is compatible with p but doesn't entail it. So, for any such c we have that $c(p|E) = 1$ (which implies that any β such that (B, β) is Jointly Almost Lockean Complete is such that $p \in \beta_E$). Now, just pick some β that is Almost Lockean Complete but $p \notin \beta_E$.

Qualitative Dutch Strategy Theorem: (B, β) avoids two-way strong dutch-bookability at threshold t iff (B, β) is jointly Almost Lockean Complete at threshold t .

Proof: We begin by defining a $E_k \in \epsilon$ conditional two-sided betting matrix \mathbf{A}_{E_k} for β_{E_k} with a fixed Lockean threshold t (in which the unconditional bets on B are given by taking E_k as the tautology). We define:

$$a_{i,j}^{E_k} = \begin{cases} 1-t & \text{if } w_i \in E_k \text{ and } p_j \in \beta_{E_k} \text{ and } w_i \in p_j \\ -t & \text{if } w_i \in E_k \text{ and } p_j \in \beta_{E_k} \text{ and } w_i \notin p_j \\ -(1-t) & \text{if } w_i \in E_k \text{ and } p_j \notin \beta_{E_k} \text{ and } w_i \in p_j \\ t & \text{if } w_i \in E_k \text{ and } p_j \notin \beta_{E_k} \text{ and } w_i \notin p_j \\ 0 & \text{if } w_i \notin E_k \end{cases} \quad (1.2)$$

Let $\vec{x} \geq 0$ be the column vector of betting stakes over all the propositions for the unconditional bets. Furthermore, let $\vec{x}_{E_k} \geq 0$ be the column vector of betting stakes over all the propositions for the E_k -conditional bets. Now, given Payoff Additivity, a straightforward computation shows that (B, β) being strongly two-way dutchbookable at threshold t is equivalent to there existing $\vec{x}, \vec{x}_{E \in \epsilon} \geq 0$ st:

$$\begin{bmatrix} \mathbf{A} & \mathbf{A}_{E_1} & \dots & \mathbf{A}_{E_{|\epsilon|}} \end{bmatrix} \begin{pmatrix} \vec{x} \\ \vec{x}_{E_1} \\ \dots \\ \vec{x}_{E_{|\epsilon|}} \end{pmatrix} < 0.$$

Now, by the Farkas-Rothschild Lemma, we know that the non-existence of such stake vectors, that is, the non-dutchbookability of (B, β) , is equivalent to there existing a row vector $\vec{c} = [c(w_1), \dots, c(w_{|W|})]$ of probabilistic credences over worlds such that:

$$\vec{c} \begin{bmatrix} \mathbf{A} & \mathbf{A}_{E_1} & \dots & \mathbf{A}_{E_{|\epsilon|}} \end{bmatrix} \geq 0.$$

Now, a little computation shows that this expression is equivalent to $\exists c$ st. [if $p \in B$, then $c(p) \geq t$ and if $p \notin B$, then $c(p) \leq t$.] and if $c(E_k) > 0$, then [if $p \in \beta_{E_k}$, then $c(p|E_k) \geq t$ and if $p \notin \beta_{E_k}$, then $c(p|E_k) \leq t$]. That is, the pair (B, β) is jointly Almost Lockean Complete at threshold t . This little computation being:

The j th coordinate of $\vec{c} \begin{bmatrix} \mathbf{A} & \mathbf{A}_{E_1} & \dots & \mathbf{A}_{E_{|\epsilon|}} \end{bmatrix}$ for the \mathbf{A}_{E_k} -part is equal to $\sum_{i \in |W|} c(w_i) a_{i,j}^{E_k}$,

so if $\sum_{i \in |W|} c(w_i) a_{i,j}^{E_k} \geq 0$, then [if $p_j \in \beta_{E_k}$, then $c(p_j \cap E_k)(1-t) + c(\neg p_j \cap E_k)(-t) \geq 0$] and [if $p_j \notin \beta_{E_k}$, then $c(p_j \cap E_k)(t-1) + c(\neg p_j \cap E_k)(t) \geq 0$]. Furthermore, if $c(E_k) > 0$, we can divide these inequalities by $c(E_k)$. Finally, using the fact that c is a probability function (or, rather, can be normalized into one without disrupting the relevant inequalities), we arrive at our result.

◇²⁸.

With this theorem in hand, we could proceed to develop a dutch strategy argument in the usual way, that is, just as in the credal case.²⁹ But, instead of rehearsing this well-trodden ground, we make a few observations about the conclusion of such dutch strategy arguments.³⁰ We stress that the Qualitative Dutch Strategy Theorem cannot, by itself, provide an argument for Plan Almost Lockean Revision. Why? Just because some agent's pair (B, β) is made jointly Almost Lockean Complete at threshold t by some credence function c does not mean that c represents that agent's actual credence function, if they have one. Thus, in this sense, we see that the requirement that rational belief plans maximize expected accuracy with respect to one's actual credences is a stronger claim. This is to be expected. After all, that claim involves different types of actual doxastic attitudes while our development of Lockean betting behavior and its susceptibility to dutchbooks only required our relevant agent to have all-or-nothing beliefs and their plannings.

We now make explicit an obvious corollary of the results of this section. This corollary can be usefully deployed in developing an accuracy-dominance argument for the rational requirement of having jointly Almost Lockean Complete belief/belief-planning pairs. We develop this accuracy-dominance argument in the upcoming section. Here is that corollary:

Qualitative SSK-BP Theorem³¹: (B, β) is strongly accuracy-dominated with respect to some Fully-Additive accuracy-measure A with threshold t iff (B, β) is not jointly Almost Lockean Complete at threshold t .

Proof: Follows immediately from the Qualitative SSK Theorem and the Qualitative Dutch Strategy Theorem. ◇.

²⁸Adapting this proof to the case of evidentially-varying thresholds $t_{E \in \epsilon}$ is straightforward, just swap the unindexed t 's in \mathbf{A}_{E_k} for the relevantly indexed threshold t_{E_k} . Also, observe that a necessary condition for (B, β) to avoid two-way weak dutchbookability and weak accuracy-dominance is for $E \in \beta_E$ and β_E to be Almost Lockean Complete for every $E \in \epsilon$. Finally, it can be analogously shown that (B, β) is not one-way strongly dutchbookable at threshold t iff (B, β) is jointly Lockean Compatible at threshold t : just keep $a_{i,j}^{E_k}$ the same as above except for setting $a_{i,j}^{E_k} = 0$ if $[w \notin E_k \text{ or } p_j \notin \beta_{E_k}]$. See (Rothschild, 2021) for the notion of Lockean Compatibility.

²⁹See (Christensen, 1996) and (Pettigrew, 2020) for such a development.

³⁰These observations also apply to the conclusion of our subsequent accuracy-dominance argument.

³¹SSK again refers to (Schervish, Seidenfeld, and Kadane, 2009). BP refers to (Briggs and Pettigrew, 2018). We note that the result found in the Briggs-Pettigrew paper follows from a special case of SSK's Corollary 2 in conjunction with the Greaves-Wallace Theorem.

1.9 A Newly Conceptualized Accuracy-Dominance Argument

It is worth stressing that the accuracy-dominance argument that we could develop for (joint) Almost Lockean Completeness from the Qualitative SSK-BP Theorem is different from the usual accuracy-dominance arguments for Probabilism and Plan Conditionalization. Whereas the latter arguments show that failing to satisfy Probabilism or Plan Conditionalization opens one up to strong accuracy-dominance for every “legitimate” accuracy-measure, the former does not. What it shows is that there exists a “legitimate” qualitative accuracy-measure such that one’s Almost Lockean Incomplete beliefs/belief-plannings perform poorly in the sense that they are strongly accuracy-dominated.³² I am inclined to think this enough to establish one’s irrationality. But how is this possible? Pettigrew (2016), for the credal case, has helpfully listed and investigated different ways of understanding the notion of “legitimacy” for accuracy-measures and the proper formulation of the relevant accuracy-dominance condition essential to all accuracy-dominance arguments. This list consists of exactly three positions: Epistemicism, Supervaluationism, and Subjectivism. Here we claim that this list is incomplete. That is, we outline a newly conceptualized accuracy-dominance principle; one that allows the usual accuracy-dominance arguments for Probabilism and Plan Conditionalization to go through as well as allowing both Rothchild’s argument and our new argument for (joint) Almost Lockean Completeness.

Here is the idea. If we understand “legitimate” as “permissible to evaluate with”, then it seems bad for there to be a manner of evaluation with which you perform relatively poorly. More precisely, it seems bad to be able to permissibly assess yourself in such a way that you perform relatively poorly, at least when there exists a way to guarantee avoiding such a bad evaluation by having different doxastic attitudes. We might call this the Evaluationist View of understanding what we mean when we impose properties on accuracy-measures. More precisely, the proposed norm is this:

Evaluationist Non-Vacuous Dominance: doxastic attitude D is irrational if

- (1): There exists a legitimate accuracy-measure A and there exists a D' which strongly accuracy-dominates D according to A . And,
- (2): There exists a D' such that D' is not strongly accuracy-dominated according to any legitimate accuracy-measure A .

A few remarks: Condition (2) is a non-vacuity condition. It helps to ensure that some doxastic attitude cannot be assessed relatively poorly in a legitimate way. After all, if every doxastic attitude is accuracy-dominated according to some legitimate manner of evaluation, then it hardly seems reasonable to flag every attitude as (equally) irrational³³. Finally, it is worth keeping in mind that, according to **Evaluationist Non-Vacuous Dominance**, (1) and (2) are offered as a sufficient

³²I thank Professor Pettigrew for suggesting that I address this matter.

³³In this way, our proposed dominance norm addresses (Pettigrew, 2016)’s concerns in the “Name Your Fortune” decision problem without going all the way to his Undominated Dominance Principle.

condition for irrationality and not as a necessary condition³⁴. It says nothing about the rational status of situations in which condition (2) fails, if there be such. Having developed our new dominance principle, we now contrast it with Epistemicism and Subjectivism.³⁵

Epistemicism is the view that there exists exactly one correct accuracy-measure (perhaps relative to a context), but that we do not know which accuracy-measure it is. All we know is that the true accuracy-measure is among our collection of legitimate accuracy-measures. The weakest corresponding Epistemicist Dominance Principle is that it is a requirement of rationality to avoid being accuracy-dominated with respect to the one true accuracy-measure. My complaint against this Epistemicist View is that it needlessly exposes Epistemicists to the risk of being irrational because they don't know which qualitative accuracy-measure is the correct one. After all, why risk the irrationality of being accuracy-dominated when it could be avoided by accepting a different formulation of the relevant dominance principle?³⁶ The Evaluationist View does not come with this bug, or the wishful thinking of there being exactly one true qualitative accuracy-measure. See (Pettigrew, 2016) for further concerns regarding the Epistemicist View.

We now contrast our proposed Evaluationist View with Subjectivism. The Subjectivist understands "legitimate" as "permissible to adopt". The Subjectivist might object to the Evaluationist View by saying that this argument is not convincing to one who adopts a non-dominated accuracy-measure. My response³⁷ is that it's unclear why such an adoption confers sole normative authority to the adopted accuracy-measure, especially when it comes to matters of being in contrast to matters of doing (Konek & Levinstein, 2017). According to Evaluationism, epistemic value theory was never in the business of describing or normatively confining peoples' preferences about epistemic matters; it's in the business of evaluating their doxastic attitudes according to purely epistemic values/concepts, whether they correctly adopt such purely epistemic values or not.³⁸ After all, it doesn't seem correct to have to first learn about someone's personal epistemic preferences in order to be able to give them an evaluation of how well their doxastic attitudes are representing the world. In this way, Subjectivism also needlessly limits the scope/applicability of accuracy-dominance arguments for epistemic norms, for what if the relevant agent adopts an illegitimate accuracy-measure? Subjectivism, unlike Evaluationism, then blocks our usual accuracy-dominance arguments from applying to such an agent, and this

³⁴For this reason, it is not a good reason to reject our proposed dominance norm on the grounds that something stronger, such as avoiding being weakly-dominated, possibly under some additional conditions, is necessary for being rational.

³⁵I leave out an extended discussion of Supervaluationism because I don't have anything new to add on the topic, except to say that I'm sympathetic with (Pettigrew, 2016)'s concerns about its extremeness in the sense that it makes evaluations of rationality require unanimity of all legitimate accuracy-measures and ignores other possible, and seemingly relevant, evaluative asymmetries, such as the one made explicit in our new Evaluationist View.

³⁶A similar point has been raised in favor of money-pump arguments for preference transitivity. Of course, such arguments might fail for other reasons.

³⁷See (Pettigrew, 2016) for other concerns about the Subjectivist View.

³⁸We note that this Evaluationist understanding of epistemic value theory is unlike traditional decision theory in which the preferences one has do confer normative authority. Maybe this is just another non-problematic point of disanalogy (Konek & Levinstein, 2017).

seems unfortunate. At the very least, Evaluationism is not easily dismissed as a plausible view of epistemic value theory; a view that gets us more of what we want. Finally, in detail, our newly conceptualized accuracy-dominance argument for joint Almost Lockean Completeness at threshold t is as follows:

- (1): Qualitative Veritism (for beliefs and their plannings).
- (2): Legitimate accuracy-measures for (B, β) 's are Fully-Additive and Extensional with threshold t .
- (3): **Evaluationist Non-Vacuous Dominance.**
- (4): **Qualitative SSK-BP Theorem.**
- (5): Therefore, rational (B, β) 's are jointly Almost Lockean Complete at threshold t .

1.10 Non-Deterministic Belief Planning

In this section, we consider the case of non-deterministic all-or-nothing belief planning and develop a qualitative dutch strategy theorem for such belief/belief-planning pairs along the lines developed by (Pettigrew, 2020) for the credal case. In this spirit, consider an agent with both all-or-nothing beliefs and their possibly non-deterministic but finitely-many plannings. Never mind if our agent also has credences or credal plannings. They might; they might not. We can thus represent such an agent's doxastic state with a pair (B, β_R) with a possibly non-deterministic belief-plan $\beta_R = (\beta_{R_{E_1}}, \dots, \beta_{R_{E_{|E|}}})$ where $\beta_{R_{E_k}} = \{\beta_{R_{E_k}^1}, \dots, \beta_{R_{E_k}^{|\beta_{R_{E_k}}|}}\}$. The interpretation, under a non-deterministic dispositional reading, being that our agent might adopt any belief-set $\beta_{R_{E_k}^i}$ in $\beta_{R_{E_k}}$ in response to receiving total evidence E_k .³⁹ Now, let $R_{E_k}^i$ be the proposition that our agent actually adopts beliefs $\beta_{R_{E_k}^i}$ in response to receiving total evidence E_k . At this point, it is useful to remind ourselves that our agent has qualitative attitudes towards propositions in some set \mathcal{F} , and $R_{E_k}^i$ need not be in \mathcal{F} . Question: when can such pairs (B, β_R) be two-way strongly dutchbooked at threshold t with a book consisting only of unconditional and $R_{E_k}^i$ -conditional bets? The answer is given by the following Qualitative Non-Deterministic Dutch Strategy Theorem, but first, a few definitions:

Def: β_R is said to be Almost Lockean Complete at threshold t iff there exists a probabilistic credence function c st. for every $p \in \mathcal{F}$ if $c(R_{E_k}^i) > 0$, then [if $p \in \beta_{R_{E_k}^i}$, then $c(p|R_{E_k}^i) \geq t$ and if $p \notin \beta_{R_{E_k}^i}$, then $c(p|R_{E_k}^i) \leq t$].

Def: the pair (B, β_R) is said to be jointly Almost Lockean Complete at threshold t iff B is Almost Lockean Complete at threshold t with respect to c and β_R is Almost Lockean Complete at threshold t with respect to the same c . The idea again being that there exists a c that “works for both”.

Qualitative Non-Deterministic Dutch Strategy Theorem: (B, β_R) avoids two-way strong dutchbookability at threshold t iff (B, β_R) is jointly Almost Lockean

³⁹Note that the planning and suppositional interpretations also generalize easily to the non-deterministic case.

Complete at threshold t .

Proof: This theorem follows as a corollary from the Qualitative Dutch Strategy Theorem. The idea is to transform the possibly non-deterministic pair (B, β_R) into the deterministic pair (B, β) in which β is a function taking “evidence” $R_{E_k}^i$ to belief-set $\beta_{R_{E_k}^i}$. Now, just apply the Qualitative Dutch Strategy Theorem to our now deterministic pairing and we’re done.

◇.

1.11 Rothschild’s Question

In this section, we answer an implicit open question of (Rothschild, 2021): when does there exist a qualitative accuracy-measure such that B is weakly accuracy-dominated? Rothschild (2021) showed that B ’s being Almost Lockean Complete is not enough to avoid weak accuracy-dominance. He provides an explicit example of a collection of beliefs that is Almost Lockean Complete, but is still weakly accuracy-dominated. More recently, Hewson (2021) improves on this by showing that a belief-set being Closed-under-single-premise-Entailment is necessary for avoiding weak accuracy-dominance.⁴⁰ What this means is that if you believe something, then you should believe whatever is (properly) entailed by your belief, on pain of being weakly accuracy-dominated. After all, it would seem odd if you believed that both Bob and Alice are coming to your party tonight, but you failed to believe that Bob is coming tonight. More precisely,

Def: B is Closed-under-single-premise-Entailment iff if $p \in B$ and p properly entails q , then $q \in B$.

In this section, we will show that (Hewson, 2021)’s result about Closure-under-single-premise-Entailment follows from our general characterization of qualitative weak dutchbookability and weak accuracy-dominance. Without further ado, here are our results:

Qualitative Weak Dutchbook Theorem: B avoids two-way weak dutchbookability at threshold $t < 1$ iff B is Almost Lockean Complete with threshold $t < 1$ with respect to a regular probability function.⁴¹

Proof:

“ \Rightarrow ”: Assume B avoids two-way weak dutchbookability at threshold $t < 1$. We use the following variant of Farkas’s Lemma (Wikipedia, 2024):

⁴⁰Of course, this observation is not a complaint against Almost Lockean Completeness as a rational norm. If it is a complaint, it is a complaint against the position that Almost Lockean Completeness is the *only* rational norm for all-or-nothing beliefs, with the upshot that Almost Lockean Completeness should be supplemented with further rational norms. Norms that hopefully also have an accuracy-centered justification.

⁴¹A credence function is said to be regular when it assigns positive credence to every possible world. The reason why the theorem has the $t < 1$ restriction is because any non-tautological belief-set is weakly dutchbookable when $t = 1$. Such belief-sets are willing to buy a \$1 bet on a contingent proposition for \$1. Also, this characterization of avoiding qualitative weak dutchbookability is not entirely unexpected because, in the credal case, weak dutchbookability is only avoided when the relevant credences are regular and probabilistic.

Let \mathbf{A} be any $m \times n$ matrix, then:

$$(\exists \vec{x} \geq 0 : \mathbf{A}\vec{x} \leq \vec{b}) \iff (\exists \vec{c} \geq 0 : \mathbf{A}^T \vec{c} \geq 0 \text{ and } \vec{b}^T \vec{c} < 0).$$

Now, B 's avoiding two-way weak dutchbookability gives us the left-hand side of the above Lemma for the following collection of \vec{b} 's: $\vec{b}_i = (0, \dots, -1, \dots, 0)$ with -1 in the i th coordinate. Thus, we have a bunch of corresponding \vec{c}_i 's such that the i th coordinate of \vec{c}_i is non-zero (and thus positive) because $\vec{b}_i^T \vec{c}_i < 0$. Furthermore, the right-hand side gives us that B is Almost Lockean Complete with respect to the \vec{c}_i 's because $\mathbf{A}^T \vec{c} = ((\mathbf{A}^T \vec{c})^T)^T = (\vec{c}^T \mathbf{A})^T \geq 0 \iff \vec{c}^T \mathbf{A} \geq 0$ and this last condition is exactly the Almost Lockean Completeness condition. Finally, we note that if B is Almost Lockean Complete with respect to the \vec{c}_i 's, then it is Almost Lockean Complete with respect to convex combinations of them. Choosing a positive convex combination then gives us a regular probability function for which B is Almost Lockean Complete.

" \Leftarrow ": Assume that B is Almost Lockean Complete with respect to a regular probability function c . Suppose, for contradiction, that B is two-way weakly dutchbookable. Now, observe that c accepts every bet that B accepts because B is Almost Lockean Complete wrt. c , so if B is weakly dutchbookable, then c is weakly dutchbookable. But, because c is regular, it is not weakly dutchbookable. Thus, B is not even two-way weakly dutchbookable.

◇.

Qualitative Weak Accuracy-Dominance Theorem: B is Almost Lockean Complete with threshold $t < 1$ with respect to a regular probability function iff there does not exist a conditionally⁴² legitimate qualitative accuracy-measure such that B is weakly accuracy-dominated.

Proof:

" \Rightarrow ": We prove the contrapositive. Suppose that B is weakly accuracy-dominated by some B' with respect to qualitative accuracy-measure A . Now, suppose, for contradiction, that B is Almost Lockean Complete with threshold $t < 1$ with respect to a regular probability function c . Thus, we know that $EA(B|c) \geq EA(B'|c)$ by (a conditional version of) the Easwaran-Dorst Theorem. But, c 's regularity and B 's being weakly accuracy-dominated by B' , that is: $A(B', w) \geq A(B, w)$ for every $w \in W$ and $A(B', w) > A(B, w)$ for some $w \in W$, implies that $A(B', w)c(w) \geq A(B, w)c(w)$ for every $w \in W$ and $A(B', w)c(w) > A(B, w)c(w)$ for some $w \in W$. Which, when summed over worlds, gives us that $EA(B'|c) > EA(B|c)$, a contradiction.

" \Leftarrow ": We prove the contrapositive. Assume that B is not Almost Lockean Complete with threshold $t < 1$ with respect to a regular probability function. Then, the above Theorem gives us that B is two-way weakly dutchbookable. Now, just apply a conditional version of Rothschild's Equivalence Theorem (which generalizes to the weak case in the correct direction) and we're done.

⁴²We are here modifying our notion of legitimacy for qualitative accuracy-measures. Instead of Variable Conservativeness, we are considering a weaker claim of Conditional Variable Conservativeness: If $T_p \neq 0$, then $F_p \neq 0$ and Variable Conservativeness holds, but if $T_p = 0$, then $F_p = 0$ also and if $F_p = 0$, then $T_p = 0$. In words, if correctly believing p adds no epistemic value, then incorrectly believing p adds no epistemic disvalue and vice versa. But, if correctly believing p does add epistemic value, then incorrectly believing p adds epistemic disvalue, in a Variable Conservative way.

◇.

Theorem: If B avoids two-way weak dutchbookability at threshold $t < 1$, then B is Closed-under-single-premise-Entailment.

Direct Proof:

We prove the contrapositive. Suppose that B is not Closed-under-single-premise-Entailment. Thus, there exists a p and q such that p properly entails q , $p \in B$, and $q \notin B$. So, our agent is then willing to buy a \$1 bet on p for \$ t and sell a \$1 bet on q for \$ t . These bets constitute a weak dutchbook against B . See the following table:

Dutchbook	$p \& q$	$\neg p \& q$	$\neg p \& \neg q$	
Payoff on p	$1-t$	$-t$	$-t$	
Payoff on q	$t-1$	$t-1$	t	
Total Payoff	0	-1	0	

◇.

Another Proof:

Suppose that B avoids two-way weak dutchbookability at threshold $t < 1$. Then, B is Almost Lockean Complete wrt. a regular probability function c , so B must be Closed-under-single-premise-Entailment. Because if not, then there exists propositions p and q such that p properly entails q and you believe p but you don't believe q , so $c(p) = c(q) = t$ which, because c is a probability function, entails c 's non-regularity.

◇.

Theorem: If there does not exist a legitimate ($t < 1$) qualitative accuracy-measure such that B is weakly accuracy-dominated, then B is Closed-under-single-premise-Entailment.

Direct Proof:

We prove the contrapositive. Assume B is not Closed-under-single-premise-Entailment. Then, the above Theorem gives us that B is two-way weakly dutchbookable at threshold $t < 1$. Now just apply the weak version of Rothschild's Equivalence Theorem and we're done.

◇.

Another Proof:

We prove the contrapositive. Assume that B is not Closed-under-single-premise-Entailment. Then, B is not Almost Lockean Complete with threshold $t < 1$ wrt. a regular probability function. Thus, Section 3's Theorem gives us that there exists a legitimate ($t < 1$) qualitative accuracy-measure such that B is weakly accuracy-dominated.

◇.

To close this section, we offer the following conjecture: B is Almost Lockean Com-

plete at threshold $t < 1$ and Closed-under-single-premise-Entailment iff B is Almost Lockean Complete at threshold $t < 1$ wrt. a regular probability function.

1.12 A Response to Hewson

In this section, we extend our results about weak dutchbookability and weak accuracy-dominance to the diachronic-ish case, that is, for belief and belief-planning pairs. In so doing, we attempt to address a concern raised by (Hewson, 2021), who shows that Closure-under-single-premise-Entailment is not necessary for belief-plannings to avoid weak accuracy-dominance. Basically, it turns out that your planned beliefs can fail to be Closed-under-single-Entailment while still avoiding being weakly accuracy-dominated for any legitimate accuracy-measure. We begin with a characterization of avoiding weak accuracy-dominance for belief and belief-planning pairs.

Qualitative Weak Accuracy-Dominance Theorem for Belief-Plannings: (B, β) is jointly Almost Lockean Complete with $t < 1$ with respect to a regular probability function iff there does not exist a conditionally legitimate accuracy-measure such that (B, β) is weakly accuracy-dominated.

Proof:

“ \Rightarrow ”: Parody the proof of the left-to-right direction of the **Qualitative Weak Accuracy-Dominance Theorem** using both the (conditional version of the) **Easwaran-Dorst Theorem** and the (conditional version of the) **Qualitative Greaves-Wallace Theorem** (along with the full-additivity of the qualitative accuracy-measure).

“ \Leftarrow ”: Suppose that there does not exist a conditionally legitimate accuracy-measure such that (B, β) is weakly accuracy-dominated. Then, (B, β) avoids weak dutchbookability, because if not, the planning version of (conditional) Rothschild’s Equivalence Theorem gives us that (B, β) is weakly dutchbookable. Now, consider the following variant of Farkas’s Lemma (Wikipedia, 2024):

Let \mathbf{A} be any $m \times n$ matrix, then:

$$(\nexists \vec{x} \geq 0 : \mathbf{A}\vec{x} \leq \vec{b}) \iff (\exists \vec{c} \geq 0 : \mathbf{A}^T \vec{c} \geq 0 \text{ and } \vec{b}^T \vec{c} < 0).$$

An entirely analogous argument to the **Qualitative Weak Accuracy-Dominance Theorem** can be made using this Lemma and the planning version of \vec{x} and \mathbf{A} as found in the **Qualitative SSK-BP Theorem**. (Observe that this argument again uses the linearity of the Lockean Condition on the right-hand-side to deal with positive convex combinations of probability functions.)

◇.⁴³

Weak Dutch Strategy Theorem for Belief-Plannings: (B, β) avoids weak dutchbookability at threshold $t < 1$ iff (B, β) is jointly Almost Lockean Complete at threshold $t < 1$ wrt. a regular probability function.

Proof:

“ \Rightarrow ”: proven in previous Theorem.

⁴³In comparison with the credal case, as found in (Nielsen, 2021), we note a trivial corollary of the above result: If there does not exist a legitimate accuracy-measure such that (B, β) is weakly accuracy-dominated, then β is Blackwell, that is, $E \in \beta_E$ for every $E \in \epsilon$.

“ \Leftarrow ”: Suppose that (B, β) is jointly Almost Lockean complete wrt. a regular probability function c . Then, (B, β) avoids weak dutchbookability at threshold $t < 1$, because, if not, then you could (by joint Almost Lockean Completeness) weakly dutchbook the (c, P) where P is the conditionalizing plan on c . But this is a contradiction because you cannot weakly dutchbook such a pair.

◇.

Armed with these results, we can, at least, partially respond to (Hewson, 2021)’s concern that avoiding weak accuracy-dominance fails to secure Closure-under-single-premise-Entailment for our planned beliefs. While we can’t get Closure-under-single-premise-Entailment, we can still get something close.

Def: β is E -Closed-under-single-premise-Entailment iff for every $E \in \epsilon$ if $p \in \beta_E$, p properly entails q , and $q - p$ contains an E -world, then $q \in \beta_E$.

Corollary: If there does not exist a conditionally legitimate accuracy-measure such that (B, β) is weakly accuracy-dominated, then β is E -Closed-under-single-premise-Entailment.

Proof: trivial.

The reason this result helps address (Hewson, 2021)’s concerns is that if your evidence requires you to discount the consideration of non- E worlds in evaluating your epistemic performance (in this case your E -planned beliefs), then it seems permissible to then discount non- E worlds in your reasoning in the sense that possessing evidence E makes certain non-logically-equivalent propositions evidentially equivalent, for you, in the sense that matters epistemically. So, perhaps, moving to E -coherence, that is, evaluating β_E only at E -worlds (as demanded by Full-Additivity), does not come with the serious *epistemic* cost that Hewson (2021) thinks it does.

1.13 Some Leitgebian Thoughts on Thresholds

So far, all of the qualitative norms that we have argued for in this paper have been parameterized by the relevant threshold t . But why think that such a threshold even exists?⁴⁴ After all, if no threshold exists, then our proposed norms seem to be useless and without content. Luckily, an idea from (Leitgeb, 2017) rescues us here. Leitgeb (2017), among other things, proposed that what makes a pair (B, c) rational is that there exists a threshold t such that the pair satisfies the (Almost) Lockean Thesis with respect to that t . A similar existentializing strategy also works for all of our qualitative norms on all-or-nothing beliefs and their plannings. In fact, we can *argue* for this position, (even in the absence of a detailed account of how t is contextually determined). The idea is that if all-or-nothing beliefs and their plannings are to play their characteristic normative roles in sanctioning bets as fair or not and in representing the world for better or worse, then such a threshold must exist because, if a threshold doesn’t exist, then your all-or-nothing beliefs and their

⁴⁴Further questions abound. Where does the threshold come from? If it is determined (or at least constrained) by contextual features, what exactly are these contextual features and how, exactly, is it so determined? Unfortunately, I do not have answers to any of these questions (if there even are answers).

plannings fail to sort permissible and impermissible gambles and they cannot be evaluated as representing the world more or less accurately. In short, if there is no threshold, then your beliefs cannot play their characteristic normative roles, but beliefs do play these characteristic normative roles, hence a threshold must exist.

1.14 Conclusion

Moral of the story: rational beliefs and their plannings are appropriately Lockean, or so says accuracy-first epistemology. This means that any account of rational beliefs and their plannings that disagrees with the relevant Lockean norms discussed in this paper is incorrect. (For example, (Pearson, 2025)’s Absolute Normal Theory of rational belief permits violations of Anticipation in contradiction to Joint Almost Lockean Completeness.) To be clear, we have not shown that further norms on your beliefs that go beyond our Lockean norms are incorrect, such as closure-under-conjunction or learning-theoretic norms (Lin, 2022) (Genin & Kelly, 2019). However, given our characterization theorems, it might be difficult to justify them from an accuracy-only standpoint, as any such justification must go beyond our proposed dominance reasoning.

To really drive home the *inescapability* of Lockeanism, at least from an Evaluationist point of view, even if we were to expand the legitimate class of qualitative accuracy-measures to include appropriately non-additive measures, only two things could happen: either the non-vacuous condition fails (in which case our proposed accuracy-dominance reasoning provides no constraints on rational belief/belief-planning pairs) or *further* constraints, constraints that imply Lockeanism, are imposed on rational belief/belief-planning pairs. So, if Evaluationist accuracy-dominance reasoning is an informative standard by which to judge the rationality of all-or-nothing beliefs and their plannings, then Lockeanism is inevitable. The only plausible pushback to this point could be that certain additive accuracy-measures are somehow *illegitimate* and necessary to evaluate non-Lockean beliefs poorly, perhaps because they fail to respect the epistemic value of informativeness or perhaps for another unknown reason entirely, but this is a question for future research.

1.15 Gestures to the Future

Future work in this area might try to further generalize our results to the case of having all-or-nothing beliefs and their plannings towards infinitely many propositions, as done in the credal case in (Kelley, ms) and citations therein. A good place to start might be looking at infinitary versions of Farkas’s Lemma. In addition, it might be desirable to develop a Qualitative General Reflection Principle in order to weaken some assumptions in our dutch-strategy/accuracy-dominance arguments along the lines of (Pettigrew, 2023) and (Staffel & De Bona, 2023). Furthermore, it seems desirable to develop guidance value arguments for our proposed qualitative norms, if possible, as done in the credal case by (Schervish, 1989) (Levinstein, 2017) (Pettigrew, 2020) (Konek, 2022). Such arguments would show how having Almost Lockean Incomplete beliefs and their plannings are bad, in a relevant sense, at guiding our actions in more general decision problems than betting scenarios.

Also, it seems desirable to develop, if possible, an accuracy-dominance argument for the Almost Lockean Thesis and Plan Almost Lockean Revision. Finally, it might be interesting to develop a notion of comparative/degrees of dutchbookability for all-or-nothing beliefs and their plannings along the lines of (Schervish, Seidenfeld, & Kadane, 2002), (De Bona & Staffel, 2017), and (De Bona & Staffel, 2021).

1.16 Appendix

1.16.1 (Shear & Fitelson, 2018)-style Argument for Plan Almost Lockean Revision

Here is the analogous Shear and Fitelson Argument:

- (1): Plan Conditionalization for credal plan $P : \epsilon \rightarrow \{\text{credence functions}\}$.
- (2): Legitimate qualitative accuracy-measures are Additive, Extensional, and Variable Conservative.
- (3): $\text{Exp}[A(\beta)|P]$ is maximized over belief plans.
- (4): Therefore, Plan Almost Lockean Revision.

We quickly show the validity of this argument:

Proof:

Condition (3) gives us that $E(A(\beta)|P) = \sum_{E \in \epsilon} \sum_{w \in E} P_E(w) A(B_E, w)$ is maximized, thus for every $E \in \epsilon$ we have that $\sum_{w \in E} P_E(w) A(B_E, w)$ is maximized. Now, because P is a conditionalization plan for one's credences c , we have that $P_E(w) = 0$ if $w \notin E$, so $\sum_{w \in E} P_E(w) A(B_E, w) = \sum_{w \in W} P_E(w) A(B_E, w) = \text{Exp}(A(B_E)|P_E)$, and, according to the Easwaran-Dorst Theorem, this latter formula is maximized when B_E satisfies the Almost Lockean Thesis with respect to P_E . Thus, we see that $E(A(\beta)|P)$ is maximized only when this latter formula is maximized, and because P_E is the conditionalization of c on E , this gives us Plan Almost Lockean Revision.

We briefly argue in favor of condition (3). This condition says that your planned credences must rationally think your planned beliefs among the epistemically best belief plans. This condition is entirely analogous to the Weak Propriety condition on credal accuracy measures and (Dorst, 2019)'s own proposed condition of maximizing the expected accuracy of one's beliefs from the perspective of one's credences.

Now, this argument is accuracy-theoretic through-and-through because we have many accuracy-only arguments for Plan Conditionalization. [As found in, say, (Greaves & Wallace, 2006), (Schervish, Seidenfeld, & Kadane, 2009), (Briggs & Pettigrew, 2018), and (Nielsen, 2021).] Still, it seems unfortunate that this argument appeals to Plan Conditionalization. After all, what if the agent doesn't even have a credal plan?⁴⁵ Thus, perhaps, we might want an accuracy argument for Plan Almost Lockean Revision without appealing to Plan Conditionalization, as provided by our expected-accuracy argument.

⁴⁵This point may not be convincing under a dispositional interpretation of credal planning. Even if so, it is better to have more arguments for a position than less, and, further, have a plausible rational requirement that is provably equivalent to our candidate norm.

1.16.2 Qualitative Diachronic Dutchbook Theorem

We begin with the promised characterization of two-way strong diachronic-dutchbookability at threshold t for Lockean bettors, but first a definition.

Def: (B_{t_1}, B_{t_2}) is said to be jointly Almost Lockean Complete at threshold t iff $\exists c$ st. B_{t_1} and B_{t_2} is Almost Lockean Complete at threshold t with respect to c . The idea again being that there exists a c that “works for both”.

Qualitative Diachronic Dutchbook Theorem: (B_{t_1}, B_{t_2}) avoids two-way strong diachronic-dutchbookability at threshold t iff (B_{t_1}, B_{t_2}) is jointly Almost Lockean Complete at threshold t .

Proof: We begin by defining our two-sided betting matrices \mathbf{A}_{t_1} and \mathbf{A}_{t_2} for B_{t_1} and B_{t_2} respectively. We define:

$$a_{i,j}^{t_k} = \begin{cases} 1-t & p_j \in B_{t_k} \text{ and } w_i \in p_j \\ -t & p_j \in B_{t_k} \text{ and } w_i \notin p_j \\ -(1-t) & p_j \notin B_{t_k} \text{ and } w_i \in p_j \\ t & p_j \notin B_{t_k} \text{ and } w_i \notin p_j \end{cases} \quad (1.3)$$

Let $\vec{x}_{t_k} \geq 0$ be the column vector of betting stakes over all the propositions for the bets given at time t_k . Now, given Payoff Additivity, a straightforward computation shows that (B_{t_1}, B_{t_2}) being two-way strongly diachronically-dutchbookable at threshold t is equivalent to there existing $\vec{x}_{t_1}, \vec{x}_{t_2} \geq 0$ st:

$$[\mathbf{A}_{t_1} \quad \mathbf{A}_{t_2}] \begin{pmatrix} \vec{x}_{t_1} \\ \vec{x}_{t_2} \end{pmatrix} < 0.$$

Now, by the Farkas-Rothschild Lemma, we know that the non-existence of such stake vectors, that is, the non-diachronic-dutchbookability of (B_{t_1}, B_{t_2}) , is equivalent to there existing a row vector $\vec{c} = [c(w_1), \dots, c(w_{|W|})]$ of probabilistic credences over worlds such that:

$$\vec{c} [\mathbf{A}_{t_1} \quad \mathbf{A}_{t_2}] \geq 0.$$

Now, a little computation shows that this expression is equivalent to $\exists c$ st. [if $p \in B_{t_1}$, then $c(p) \geq t$ and if $p \notin B_{t_1}$, then $c(p) \leq t$] and [if $p \in B_{t_2}$, then $c(p) \geq t$ and if $p \notin B_{t_2}$, then $c(p) \leq t$]. That is, the pair (B_{t_1}, B_{t_2}) is jointly Almost Lockean Complete at threshold t .

◇.

With this characterization in hand, we note a difference between the qualitative and credal case of diachronic-dutchbookability. Namely, there is more wiggle room in the qualitative case than the credal case in the sense that B_{t_1} need not equal B_{t_2} . After all, the diachronic belief pair $(\{p, T\}_{t_1}, \{T\}_{t_2})$ is jointly Almost Lockean Complete for $t < 1$, just take any $c(p) = t$. It is the “Almost”-part in Almost Lockean

Complete which opens the door to avoiding diachronic-dutchbookability while having different belief-sets at different times. Nevertheless, even with this extra wiggle room, the requirement of avoiding diachronic-dutchbooks in the qualitative case is implausibly demanding, as previously established.

Finally, for pedagogical purposes, we also include an elementary proof of one direction of the Qualitative Dutch Strategy Theorem. A straightforward proof of the other direction is still wanting.

Proposition: If (B, β) is jointly Almost Lockean Complete at threshold t , then (B, β) avoids two-way strong dutchbookability at threshold t .

Proof: (We follow the proof strategy found in (Nielsen, 2021).) Assume the antecedent. Let c be a probabilistic credence function making (B, β) jointly Almost Lockean Complete at threshold t . Suppose, for contradiction, that (B, β) is two-way strongly dutchbookable at threshold t . Given the Qualitative SSK Theorem, this implies that (B, β) is strictly accuracy-dominated for some fully-additive accuracy measure A , that is, $\exists(B', \beta')$ st. $A[(B', \beta'), w] > A[(B, \beta), w]$ for every $w \in W$. Clearly, this implies that $\text{Exp}[A(B', \beta')|c] > \text{Exp}[A(B, \beta)|c]$. But, given Qualitative Temporal Separability, $\text{Exp}[A(B, \beta)|c] = \text{Exp}[A(B)|c] + \text{Exp}[A(\beta)|c]$ and, given that B and β is Almost Lockean Complete at threshold t with respect to c , the Easwaran-Dorst Theorem and the Qualitative Greaves-Wallace Theorem imply that the pair (B, β) maximizes $\text{Exp}[A(B^*, \beta^*)|c]$ (when considered as a function from pairs (B^*, β^*)), contradicting $\text{Exp}[A(B', \beta')|c] > \text{Exp}[A(B, \beta)|c]$.
 \diamond .

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