Some Leit Commentary

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In this section, we apply our newly developed concepts and mathematical techniques to the study of the stability of all-or-nothing beliefs simpliciter. It has been argued, most prominently in (Loeb, AHHHHH) and (Leitgeb, 2017), that the stability of belief is rationally required. It is thus an interesting question to what extent dutchbook and accuracy-dominance arguments can be developed in favor of stability's rational requirement. While I ultimately do not find these arguments convincing, I think them interesting enough to include here. We begin by reviewing the inspired work of Leitgeb (2017).

Leitgeb (2017) proposed the following as a candidate norm on belief/credence pairs.[1]

Def: The Almost Humean Thesis (with threshold $t < 1$): (B, c) is such that:
(1): it satisfies the Almost Lockean Thesis (with threshold $t < 1$)[2][3].
(2): If $p \in B$, then $\forall y \in Pos(B)$, $c(p|y) \geq t$.
(3): If $p \notin B$, then $\exists y \in Pos(B)$ st. $c(p|y) \leq t$.

All of this is well and good, but notice that the Almost Humean Thesis, like the Almost Lockean Thesis, is a constraint on belief/credence pairings. So, what can we say about the relationship between stability and just all-or-nothing belief? Never mind if our agent also has credences. They might; they might not. We are interested in studying Leitgeb's notion of stability for all-or-nothing belief simpliciter. In this spirit, consider the following candidate norm.

Def: B is Almost Humean Complete (with threshold $t < 1$) iff there exists

[1] Strictly speaking, this formulation is different than Leitgeb's. Once again, we use the "Almost" modifier to stress that we are working with possibly non-strict inequalities.

[2] A proposition p is said to be belief-possible according to belief-set B, $p \in Pos(B)$, iff you don't believe not-p, that is, $\neg p \notin B$.

[3] We could choose this Lockean threshold to be different than the Humean threshold, if desired. Also, we are building the Almost Lockean Thesis into the definition of the Almost Humean Thesis, in contrast to (Leitgeb, 2017) and taking $t < 1$ in order to secure that if $y \in Pos(B)$, then $c(y) \neq 0$, at least when c is a probability function. This is beneficial because then all the conditional confidences in conditions (1) and (2) are well-defined.

a probabilistic c such that (B, c) satisfies the Almost Humean Thesis (with threshold $t < 1$).

Once again, just as in the case of Almost Lockean Completeness, c's that make your beliefs Almost Humean Complete need not represent your actual credences. Thus, even if we could successfully argue for Almost Humean Completeness, we cannot, immediately, argue for the Almost Humean Thesis. So, why think Almost Humean Completeness a requirement of epistemic rationality?

1.1 Humean Betting and Humean Dutch-strategies

The Stable bettor (with threshold t) is just like the Lockean bettor (with threshold t) when it comes to unconditional bets: If $p \in B$, then the agent will (or ought) to buy a \$$x \geq 0$ bet on p for \$$tx$ (or less). And, if $p \notin B$, then the agent will (or ought) to sell a \$$x \geq 0$ bet on p for \$$tx$ (or more). It's when it comes to conditional bets that they differ. The Lockean bettor (with just all-or-nothing beliefs) rejects (or takes as impermissible) all (non-tautological) conditional bets. The Stable bettor, on the other hand, accepts some conditional bets: If $p \in B$, then the agent will (or ought) to buy a \$$x \geq 0$ y-conditional bet on p for \$$tx$ (or less) given that the condition y is belief-possible. Furthermore, we say that a Stable bettor is $\vec{y}$-Humean if she accepts the following scheme of conditional bets: if $p \notin B$, then the agent will (or ought) to sell a \$$x \geq 0$ y_p-conditional bet on p for \$$tx$ (or more) such that $y_p \in Pos(B)$. Finally, we say that B is strongly Humean dutchbookable if B, as a $\vec{y}$-Humean bettor, is strongly dutchbookable for every $\vec{y}$. It seems desirable for your belief-set B to avoid strong Humean dutchbooks. So, what does B have to look like in order to avoid strong Humean dutchbooks?

Qualitative Humean Dutchbook Theorem: B avoids strong Humean dutchbooks (with threshold $t < 1$) iff B is Almost Humean Complete (with threshold $t < 1$).

Proof: We begin by defining a $y_k \in Pos(B)$ conditional betting matrix $\boldsymbol{A}_{y_k}$ with a fixed threshold t (in which the unconditional bets on B are given by taking y_k as the tautology τ) and a $y_{p_l} \in Pos(B)$ conditional betting matrix $\boldsymbol{A}_{y_{p_l}}$ with a fixed threshold t. We define:

$$a^{\tau}_{i,j} = \begin{cases} 1-t & p_j \in B \text{ and } w_i \in p_j \\ -t & p_j \in B \text{ and } w_i \notin p_j \\ -(1-t) & p_j \notin B \text{ and } w_i \in p_j \\ t & p_j \notin B \text{ and } w_i \notin p_j \end{cases} \tag{1}$$

$$a_{i,j}^{y_k} = \begin{cases} 1-t & \text{if } w_i \in y_k \text{ and } p_j \in B \text{ and } w_i \in p_j \\ -t & \text{if } w_i \in y_k \text{ and } p_j \in B \text{ and } w_i \notin p_j \\ 0 & \text{if } w_i \in y_k \text{ and } p_j \notin B \text{ and } w_i \in p_j \\ 0 & \text{if } w_i \in y_k \text{ and } p_j \notin B \text{ and } w_i \notin p_j \\ 0 & \text{if } w_i \notin y_k \end{cases} \tag{2}$$

$$a_{i,j}^{y_{p_l}} = \begin{cases} 0 & \text{if } w_i \in y_{p_j} \text{ and } l = j \text{ and } p_j \in B \text{ and } w_i \in p_j \\ 0 & \text{if } w_i \in y_{p_j} \text{ and } l = j \text{ and } p_j \in B \text{ and } w_i \notin p_j \\ -(1-t) & \text{if } w_i \in y_{p_j} \text{ and } l = j \text{ and } p_j \notin B \text{ and } w_i \in p_j \\ t & \text{if } w_i \in y_{p_j} \text{ and } l = j \text{ and } p_j \notin B \text{ and } w_i \notin p_j \\ 0 & \text{if } w_i \notin y_{p_j} \text{ or } l \neq j \end{cases} \tag{3}$$

Let $\vec{x} \geq 0$ be the column vector of betting stakes over all the propositions for the unconditional bets. Furthermore, let $\vec{x}_{y_k} \geq 0$ be the column vector of betting stakes over all the propositions for the y_k-conditional bets. Let $\vec{x}_{y_{p_l}} \geq 0$ be the column vector of betting stakes over all the propositions for the y_{p_l}-conditional bets. Now, given Payoff Additivity, a straightforward computation shows that B being strongly $\vec{y}$-Humean dutchbookable (with threshold $t < 1$) is equivalent to there existing $\vec{x}$,$\vec{x}_{y \in Pos(B)}$,$\vec{x}_{y_{p \notin B}} \geq 0$ st:

$$\begin{bmatrix} \boldsymbol{A}_\tau & \boldsymbol{A}_{y_1} & ... & \boldsymbol{A}_{y_{|Pos(B)|}} & \boldsymbol{A}_{y_{p_1}} & ... & \boldsymbol{A}_{y_{p_{|p \notin B|}}} \end{bmatrix} \begin{pmatrix} \vec{x} \\ \vec{x}_{y_1} \\ ... \\ \vec{x}_{y_{|Pos(B)|}} \\ \vec{x}_{y_{p_1}} \\ ... \\ \vec{x}_{y_{p_{|p \notin B|}}} \end{pmatrix} < 0.$$

Now, by the Farkas-Rothschild Lemma, we know that the non-existence of such stake vectors, that is, the non-dutchbookability of $\vec{y}$-Humean B, is equivalent to there existing a row vector $\vec{c} = [c(w_1), ..., c(w_{|W|})]$ of probabilistic credences over worlds such that:

$$\vec{c} \begin{bmatrix} \boldsymbol{A}_\tau & \boldsymbol{A}_{y_1} & ... & \boldsymbol{A}_{y_{|Pos(B)|}} & \boldsymbol{A}_{y_{p_1}} & ... & \boldsymbol{A}_{y_{p_{|p \notin B|}}} \end{bmatrix} \geq 0.$$

Now, a little computation shows that this expression is equivalent to $\exists c$ st. (B, c) satisfies the Almost Humean Thesis (with threshold t) with the relevant y's in condition (2) being the y_{p_l}'s.[4]

[4]As a historical note: observing that, mathematically, the Humean Thesis imposes inequality constraints on credences and knowing that the Farkas approach handles these kinds of inequalities quite well is what first led me to investigate the stability of all-or-nothing beliefs

$\diamond$.

Using this result, we can, in analogy with the Lockean case, develop a dutchbook argument for Almost Humean Completeness.

(1): Humean Betting (with threshold $t < 1$).
(2): Payoff Additivity.
(3): Avoidance of strong Humean dutchbooks (with threshold $t < 1$).
(4): **Qualitative Humean Dutchbook Theorem**.
(5): Therefore, Almost Humean Completeness (with threshold $t < 1$).

Unfortunately, this argument is no more convincing than in the Lockean case. For one thing, Humean betting includes Lockean betting, which we already found to be rather ad hoc. The reason that we detailed this Humean dutchbook argument, in contrast to avoiding such a detailing in the Lockean case, is because, despite its lack of persuasive appeal, it is still better than the corresponding accuracy-dominance argument. In more detail, we can prove the following:

Humean Equivalence Theorem: B is strongly Humean dutchbookable (with threshold $t < 1$) iff for every $\vec{y} = (y_{p_1}, ..., y_{p_{|p \notin B|}})$ there exists a B' and a legitimate accuracy-measure A such that $A(B, w) + \sum_{y \in Pos(B)} A_y(B, w) < A(B', w) + \sum_{p_l \in B'} a_{y_{p_l}}(p_l \in B', w)$ for every w .

Proof: omitted. (Follows in analogy with Rothschild's Equivalence Theorem.)

While this result, in conjunction with the former, could be used to mount some kind of accuracy-dominance argument for Almost Humean Completeness, it is unclear that such an argument would be convincing. After all, why is it good to avoid accuracy-dominance of the form found in the **Humean Equivalence Theorem**? In some sense, this concern is to be expected. After all, we proved that avoiding accuracy-dominance, in the usual sense, is equivalent to being Almost *Lockean* Complete. So, minus a plausible motivation for accepting the Humean form of accuracy-dominance, this argument, along with the dutchbook argument given above, is no more convincing than the stability condition itself.[5]

At this point, we could extend our results to the cases of avoiding weak Humean dutchbooks/accuracy-dominance and the corresponding belief-planning cases, but, given the remarks above, we expect to find these arguments equally, if not more, unconvincing. With this in mind, we move on to consider other topics of interest.

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using our Farkas approach.

[5]A similar point, given Dorst's (2019) expected-accuracy argument for the Almost Lockean Thesis, can be raised against Leitgeb's sufficientist expected-accuracy argument for the Almost Humean Thesis found in chapter 5 of (Leitgeb, 2017).